Wave Motion in 1D

*Light is a transverse wave.

- 1D travelling: $\psi = f(x,t) = f(x \mp vt)$
  - $x - vt$ to right
  - $x + vt$ to left

  ↳ At a particular time, t, $\psi(x)$ is a wave profile

- For 1D travelling wave:

  ① In form of $f(x \pm vt)$

  ② $$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

  ③ $$\frac{\partial x}{\partial t} = v = \frac{-\partial \psi/\partial t}{\partial \psi/\partial x}$$

  If this ratio is neg, in $-x$ dir.

- Harmonic wave:

  amplitude

  $$\psi = A\cos(kx \mp \omega t \pm \varepsilon) = A\cos(k(x \pm vt) \mp \varepsilon)$$

  phase.

  or $$\psi = A e^{i(kx \mp \omega t \pm \varepsilon)}$$

  $$e^{i\theta} = \cos\theta + i\sin\theta.$$

- Parameters:

  wave number.

  $$\tau = \frac{\lambda}{v}, \quad k = \frac{2\pi}{\lambda}$$

  $$f = \frac{1}{\tau} = \frac{v}{\lambda},$$

  $$\omega = 2\pi f = kv \Rightarrow v = \frac{\omega}{k} = \lambda f.$$

  angular frequency

  ↳ phase velocity

- Superposition:

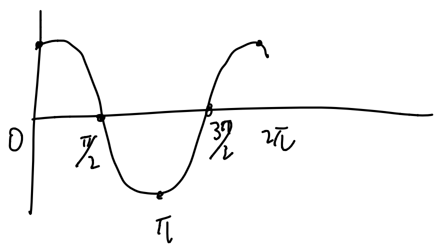
$$\psi_T = \psi_1 + \psi_2 + \psi_3 + \dots$$

- Fourier:

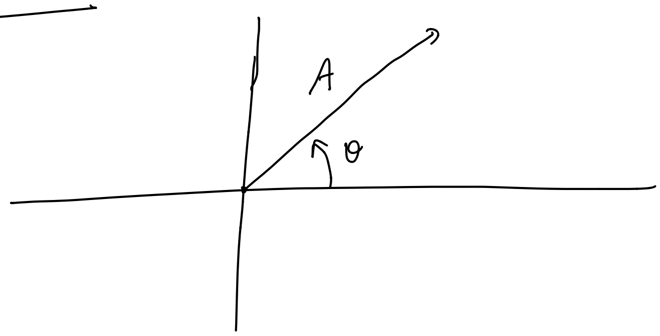
all harmonic

$$\psi = A_1 \sin_1 + A_2 \sin_2 + \dots + A_{n+1} \cos_1 + A_{n+2} \cos_2 + \dots$$

- Constructive: $\varepsilon = 0,\ 2\pi$ destructive: $\varepsilon = \pi/2$



- Phasors:



$$\varphi = A \cos(\underbrace{kx + \omega t + \varepsilon}_{\theta})$$

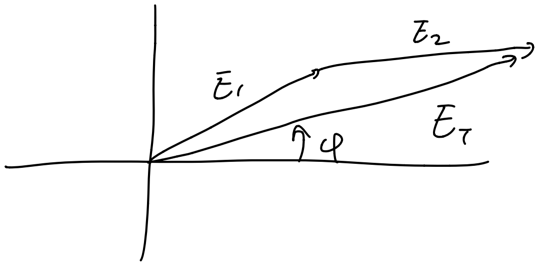
- Adding 2 waves:

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$

$$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2}$$

$$\varphi_T(x,t) = E_0 \sin(kx - \omega t + \alpha)$$

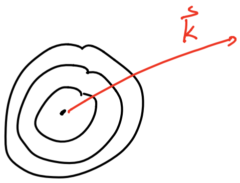
- Adding waves with phasors:



$E_T = E_1 + E_2$

- 3D Wave Motion:

  - 2D waves



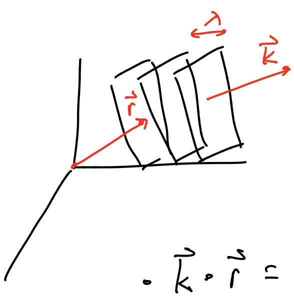
  - $E \propto \frac{1}{r}$

  - direction of propagation is $\vec{k}$

    - $|\vec{k}| = \frac{2\pi}{\lambda}$.

    - $\vec{k} \perp$ wave fronts $\Rightarrow$ only in $\hat{r}$

  - Planar Wave



- $\vec{k} \cdot \vec{r}$ = constant. $= k r_k$.

$\Rightarrow \varphi = f(\vec{k} \cdot \vec{r} - \omega t + \varepsilon)$

denotes direction of travelling.

- ex: $\varphi(\vec{r}, t) = \vec{A} e^{i(k r_k \pm \omega t)}$

and $\vec{v} = \frac{\omega}{k} \hat{k}$

- Polarization of EM Wave

  - For EM wave, $\vec{E}$ & $\vec{B}$ travels in the direction of $\hat{k}$
  - yet, $\vec{E}$ and $\vec{B}$ are $\perp$ in the oscillation

    - $\vec{E} \perp \vec{B} \perp \vec{k} \Rightarrow \boxed{\begin{array}{l} \vec{E}\cdot\vec{B}=0,\ \vec{E}\cdot\vec{k}=0,\ \vec{B}\cdot\vec{k}=0 \\ \vec{E}\times\vec{B} = |E|\,|B|\,\hat{k}. \end{array}}$

    - Ex: if $\vec{k}$ is in x, then:

      ossilation direction

      $$E_y(x,t) = E_{o_y} \sin(kx-\omega t+\varepsilon)$$

      direction of travelling

      $$B_z(x,t) = B_{o_z} \sin(kx-\omega t+\varepsilon)$$

      and $\vec{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ E_y \\ 0 \end{bmatrix}$, $|B_o| = \frac{1}{c}|E_o|$

      $\vec{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ B_z \end{bmatrix}$

  - Linearly polarized if $\vec{E}$ orientation is constant.

- Velocity of EM wave in vacuum:

  $$v = c = 3.0\times10^{8}\ m/s = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

- Velocity of EM wave in a medium

  $$n = \frac{c}{v} = \frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_o\mu_o}}$$

  $$\Rightarrow \lambda = \frac{\lambda_o}{n}$$

  $$v = \frac{c}{n}, \quad \underline{\lambda f = v}$$

- Spherical Wave

  - produce by a point source, uniform in all direction.

  - $\varphi(\vec{r},t) = \vec{E}_0(r)\, e^{i(\vec{k}\cdot\vec{r} \pm \omega t + \varepsilon)} = \frac{\vec{A}}{r} e^{i(\vec{k}\cdot\vec{r} \pm \omega t + \varepsilon)}$

    - $\vec{A} = \vec{E}_0(1) \Rightarrow$ source strength.

    - $\boxed{|\vec{E}_0| \propto \frac{1}{r}}$

- Properties of EM Waves

  - Poynting Vector ($\vec{S}$)

    - Energy flux density $\longrightarrow \frac{\text{Energy}}{\text{time} \times \text{area}}$.

    - $u_E = \frac{\varepsilon_0}{2} E^2$, $u_B = \frac{1}{2\mu} B^2$

      $\therefore \boxed{S = uc} = c\varepsilon_0 E^2 = \frac{1}{\mu_0} EB$

      $\Rightarrow$ Energy density in a EM wave: $u_E + u_B = \frac{1}{\mu_0} B^2 = \varepsilon_0 E^2$

      $u = \frac{E}{V}$.

  - Irradiance (I) / radiant flux density

    - average flow of energy: $I = \langle \vec{S} \rangle_T = \frac{1}{T}\int_0^T S(t)\,dt$.

    - amount of light illuminating a surface

      - avg E/area/time

    - $\boxed{I = \frac{1}{2} c\,\varepsilon_0 E_0^2} \Rightarrow I \propto E_0^2$ → inverse square law.

      and $\boxed{I \propto \frac{1}{r^2}}$ → since area is a sphere from a point source

$$\circ\; I = \frac{\phi}{A} \underbrace{hf}_{\text{Energy per photon}}$$

\# of photon/s

area

- ϕ = photon flux = photon / sec

- $I = \frac{\text{Power}}{\text{Area}} \Rightarrow \boxed{\text{power} = IA}$

- Pressure / Energy density

  - Pressure = energy density $= \frac{E}{\text{Volume}} = \frac{F}{\text{Area}} = \frac{\text{Work}}{\Delta d A}$.

$\Rightarrow \boxed{P = u = \frac{I}{c}}$ for absorbing surface (avg).

$\boxed{P = \frac{2I}{c}}$ for reflecting surface (avg)

} both in $\hat{k}$ dir.

$P = \frac{S}{c}$ instaneous for an absorbing surface.

- Force ($\vec{F}$)

  - $\vec{F} = \vec{P}_{\text{ressure}} \times A = \frac{E}{\Delta d}\hat{k} = \frac{\Delta p}{\Delta t}$

  $\therefore \vec{F} = \frac{2IA}{c}\hat{k} = \frac{2\,\text{Power}}{c}\hat{k}$

- Momentum ($\vec{P}$)

$$\Delta \vec{p} = \vec{F}\Delta t$$

momentum density ($\vec{P}$ per unit volume)

Pressure $\vec{k}$

$$\vec{P}_v = \frac{\vec{P}}{A c \Delta t} = \frac{\vec{F}}{Ac} = \frac{I}{c^2}\hat{k} = \frac{u}{c}$$

• Photons

- $E_p = hf$ → energy per photon
- $E_{total} = Nhf$ → total energy for N photons
- $I = \frac{Power}{Area} = \frac{Energy}{time \cdot Area}$ Irradiance of photon (or intensity) (or flux density)

  $\Rightarrow N \propto I$

- $c = \lambda f$ in vacuum, or $v = \lambda f = \frac{c}{n}$

- Photoelectric Effect:
  - $E_k = hf - e$, e = threshold energy
  - $1 \, eV = 1.602 \times 10^{-19} \, J$

- photon flux $(\phi) = \frac{Power}{hf} = \frac{photons}{sec.}$

- Momentum $(p) = \frac{hf}{c} = \frac{E}{c}$

- Force $(F) = \frac{-\Delta p}{\Delta t} = -\phi \Delta p$
  - For absorbing surface: $\Delta p = -P_{photon}$, $F = \frac{Power}{c}$
  - For reflecting surface: $\Delta p = -2P_{photon}$, $F = \frac{2 \, Power}{c}$

- Mean photon density:

$$\frac{\phi}{A} = \frac{\text{Power}}{A h f} = \frac{I}{hf}$$

- Pressure: $\left(\frac{F}{A} = \frac{-\Delta P}{A \Delta t} = \frac{\Delta E}{4 c \Delta t} \right)$ or energy density

$$\text{Pressure} = \frac{\text{Power}}{cA} = \frac{I}{c} \quad \text{for \underline{absorbing}}$$

$$\text{pressure} = \frac{2\,\text{Power}}{cA} = \frac{2I}{c} \quad \text{for \underline{reflecting}}$$

Light Matter Interaction

- Rayleigh Scattering $\underline{\underline{\propto \lambda_0^{-4}}}$
  - Blue sky / Red Sunset due to:
    - More blue perpendicular to travel
    - less blue parallel to travel
  - For particles $<< \lambda_0$
- Mie Scattering
  - Large particles scatter more at longer λ then small particles
  - particle <u>similar</u> size to λ → scatter is <u>λ independent</u> → looks white

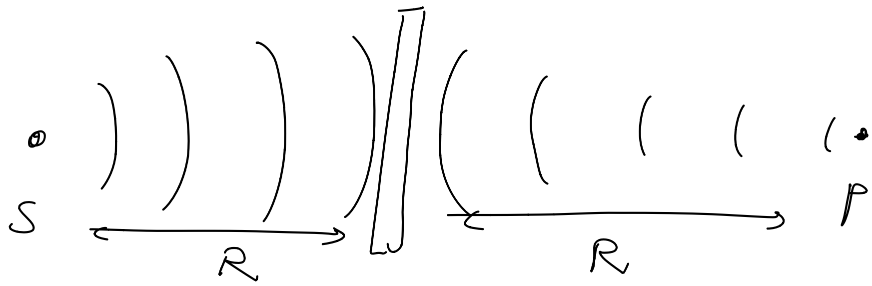
- Transparency : $\longrightarrow$ lack of scattering
  - short range order
  - distance b/w molecules $\sim \lambda/2$
  - E is transmitted, not scattered

- Dispersion :
  - n is a function of λ
  - n changes most rapidly at resonant frequency.
    - $n(\lambda_0)$, $n(\omega)$ $\Rightarrow$ $\omega\uparrow$, $n\uparrow$; $\lambda\uparrow$, $n\downarrow$
      - $\hookrightarrow$ $f(\omega)\downarrow$, $n(\omega)\uparrow$
  - $n_{gas} < n_{solids}$

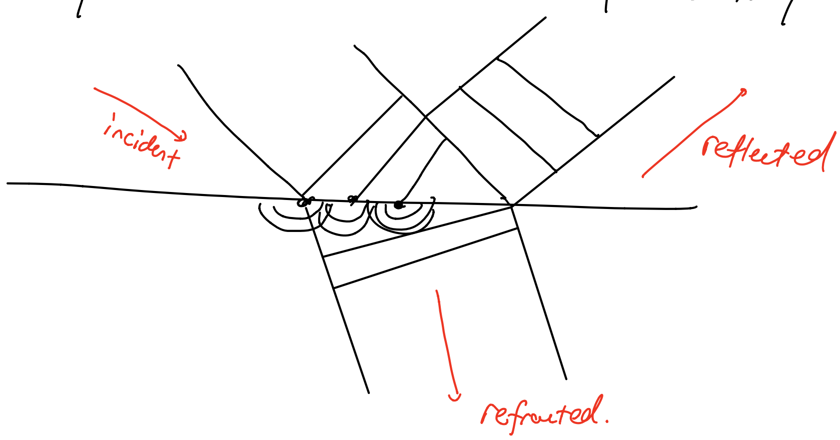
- Types of Interaction
  - Transmission (including refraction)
  - Scattering (oscillation of dipoles)
  - Reflection (a type of scattering)
  - Absorption (with or with-out re-emission)

Light Interactions at Interfaces

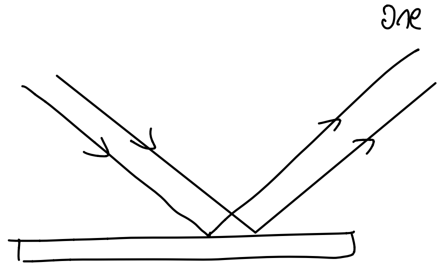
- Ray model of light
    - radius of curvature: distance of wavefront to object



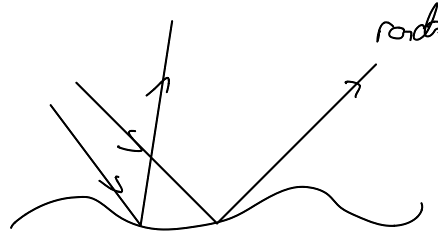
- Based on Huygen's principle:
    - Every point is a source of secondary wave front.



- Specular reflection:



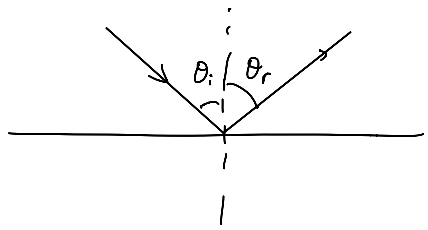
- Diffuse Reflection:



- Law of Reflection

reflected angle = incident angle

$\theta_r = \theta_i$



- Law of refraction

  - $n_i \sin\theta_i = n_t \sin\theta_t$ → Snell's Law
    ↳ Reversibility of Rays.

$$\frac{f_i}{f_t} = \frac{n_t}{n_i}, \quad \omega_r = \omega_t$$

  - $$\frac{\lambda_i}{\lambda_t} = \frac{n_t}{n_i}$$

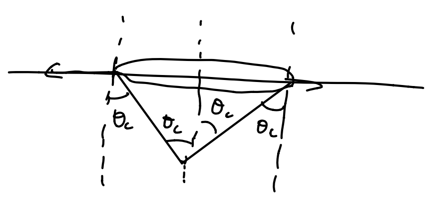
- Pencil : all rays from a point object

  Beam : all pencils from an extended object

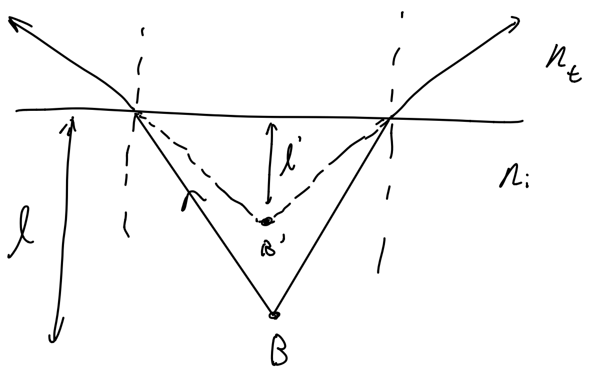
- Total Internal Refraction (TIR)

$\theta_i = \theta_c$ if $\theta_t = 90°$

$\Rightarrow \theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right)$



- **Apparent Depth**

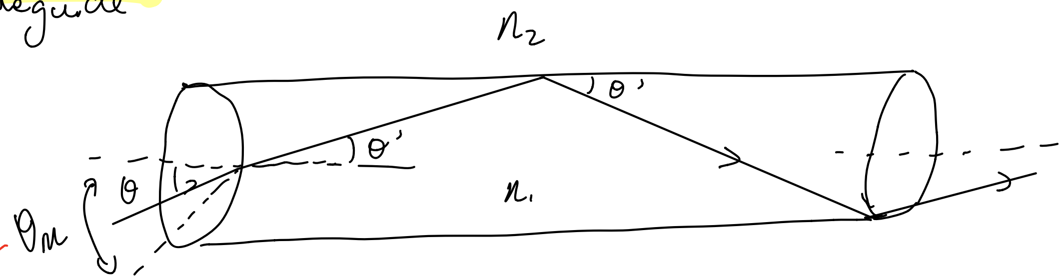


$$\boxed{l' = \frac{-n_t}{n_i} l}$$

$\Rightarrow$ virtual image if $l' < 0$

real object if $l > 0$

- **Waveguide**



Limiting angle captured by the fibre

$n_2 \sin\theta_m = n_1 \sin\theta_m'$

- $N.A = n_1 \sin\theta_m = (n_1^2 - n_2^2)^{1/2} \rightarrow$ **Numerical Aperture**

  ↳ max cone of light captured & guided.

- $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

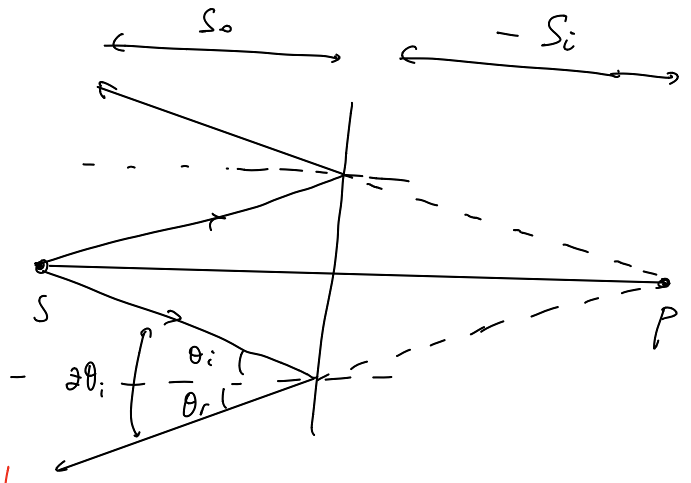
- $\theta_m' = 90 - \theta_c$

  ↳ $\theta_m = \sin^{-1}\left(\frac{n_1}{n_2} \sin\theta_m'\right) \Rightarrow$ cone angle $= 2\theta_m$.

- If $\theta_i > \theta_c$, it is reflected into the tube

  If $\theta_i < \theta_c$, it is partially refracted out.

Mirrors



- $\theta_i = -\theta_r$

$$\frac{1}{S_o} + \frac{1}{S_i} = 0$$

S_o is real (>0)

S_i is virtual (<0)

* Technically, $\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f_o}$, but $f_o = \infty$ for reflection, since $f_o = \frac{-R}{2}$

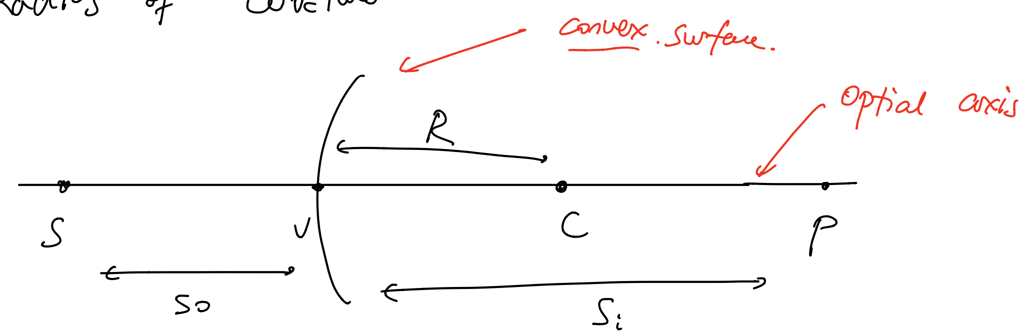
- Transverse (height) magnification:

$$M_T = \frac{y_i}{y_o} = \frac{-S_i}{S_o} = 1 \Rightarrow \text{same height for plane mirror}$$

$\Rightarrow$ Image is erect (upright) & reversed left to right.

Refraction Surfaces

- Radius of Curvature



- $R > 0$ for Convex
- $R < 0$ for concave

- Vergence $= \frac{n}{s}$, Δ vergence $= \mathcal{D}$ = power of surface.

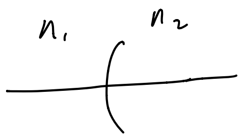
- Sign conventions: Assume light goes from left to right.
  - S_o, $f_o > 0$ at left of V
  - S_i, $f_i > 0$ at right of V
  - $R > 0$ at right of V
  - y_o, $y_i > 0$ at up of optical axis

- $$\boxed{\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{(n_2 - n_1)}{R}} = \mathcal{D} \quad (\text{in } m^{-1})$$

- Focal points and Focal length:
  - F_o : first focal point $\Rightarrow$ object point w/ $S_i = \infty$

    ↳ distance from V to F_o = f_o
  - F_i : 2^{nd} focal point $\Rightarrow$ image point w/ $S_o = \infty$

    ↳ distance from V to F_i = f_i

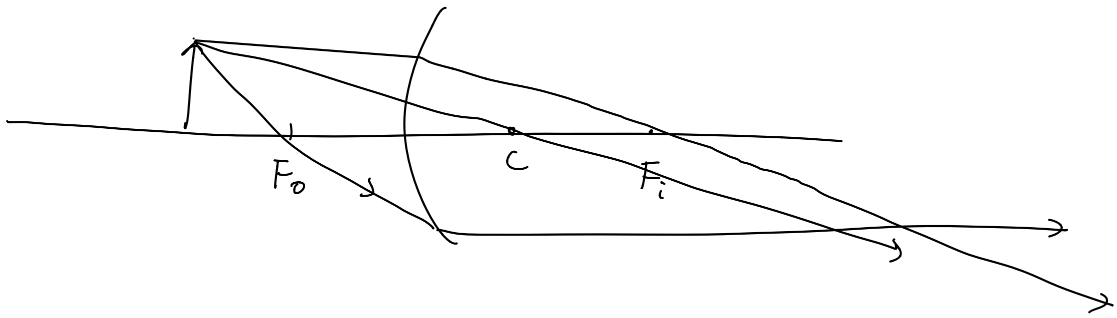


$\Rightarrow$

$$\boxed{\begin{gathered} \frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_1}{f_o} = \frac{n_2}{f_i} = \mathcal{D}. \qquad f_o - f_i = -R \\ f_i = \frac{n_2 R}{(n_2 - n_1)}, \quad f_o = \frac{n R.}{(n_2 - n_1)} \qquad \frac{f_i}{f_o} = \frac{n_2}{n_1} \\ CF_i = f_i - R = f_o, \quad CF_o = R + f_o = f_i \end{gathered}}$$

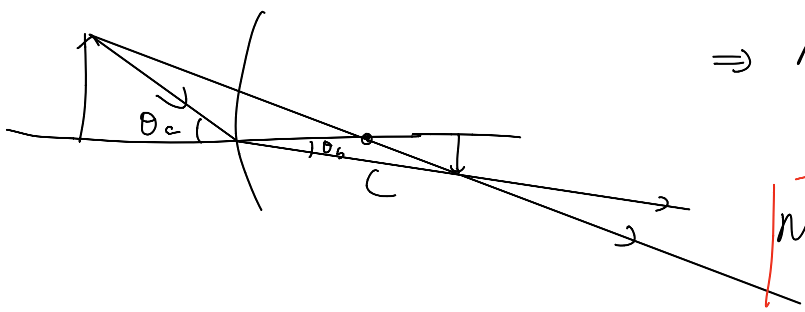
- **Ray tracing:** (on axis)

  1. Through F_o $\rightarrow$ // to optical axis
  2. Parallel to axis $\rightarrow$ through F_i
  3. Undeviated if through C



- $M_T = \dfrac{y_i}{y_o}$

$y_i = -S_i \tan\theta_b$, $y_o = S_o \tan\theta_a$

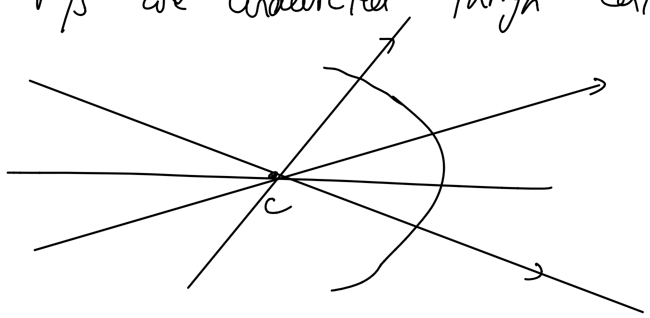


$$\Rightarrow M_T = \frac{-S_i \tan\theta_b}{S_o \tan\theta_a}$$

$$\boxed{M_T \approx \frac{-S_i n_1}{n_2 S_o}}$$

for small angle

- **Off axis imaging:**
  - nodal rays are undeviated through centre of curvature



use angular magnification (mm/deg)

Thin Lenses

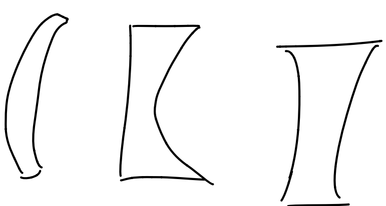
- 2 spherical interfaces with <u>small</u> seperation
  - $S_{o_2} = -S_{i_1}$ → Thin lens approximation

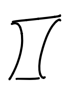
- Thin Lens Equation:

$$\frac{n_m}{S_o} + \frac{n_m}{S_i} = (n_L - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \mathcal{D}.$$

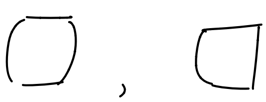
* S_o & S_i are measured from lens' <u>centre</u>.

- <u>Types of Thin Lens (assume $n_L > n_m$)</u>
  - −ve power lens ⇒ Concave



  - Biconcave: $R_1 < 0$, $R_2 > 0$ 
    - include: equal concave: $-R_1 = R_2$

  - +ve power lens → Convex.

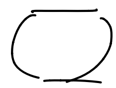


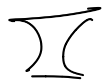
  - Biconvex + equal convex

- Focal Points :

- $$\boxed{\frac{n_m}{f} = (n_L - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \quad f = f_o = f_i$$

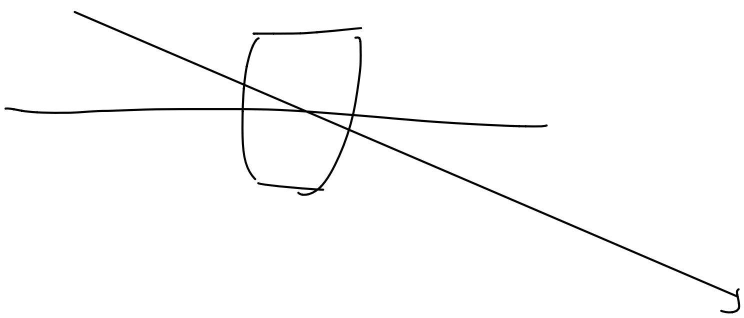
AND $$\frac{n_m}{S_o} + \frac{n_m}{S_i} = \frac{n_m}{f}$$

- for converging / convex lens : $f > 0$ 

  for diverging / concave lens : $f < 0$ 

- Ray Tracing :

  - Nodal ray : undeviated through centre of lens

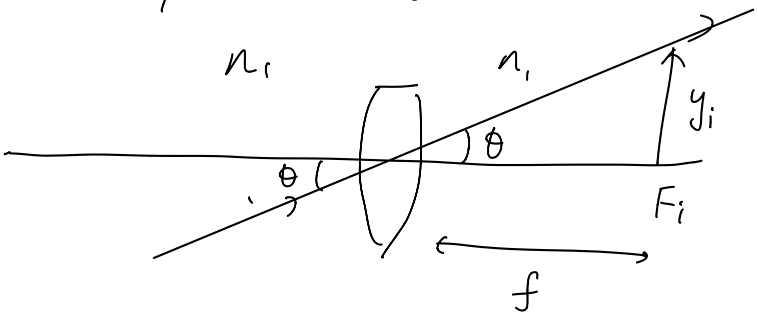


① Through F_o ⇒ // to axis

② // to axis ⇒ through F_i

③ through Centre ⇒ undeviated.

- Off axis / Distance object:



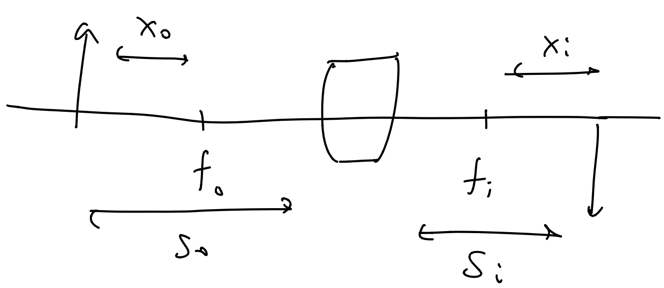
$$y_i = -f \tan\theta.$$

- Magnification (Transversal)
  - $M_T = \dfrac{-S_i}{S_o}$ for thin lens
  - Properties changes as S_o is changed
    - ↳ see thin lens pg. 42.

- Newtonian Formula

$S_o = x_o + f_o \Rightarrow x_o$ +ve if left of F_o

$S_i = x_i + f_i \Rightarrow x_i$ +ve if right of F_i

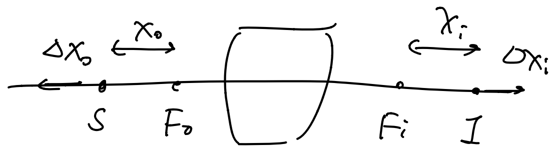


$x_o x_i = f_o f_i \xRightarrow{\text{For Thin lens}} x_o x_i = f^2$

$$\therefore M_T = \frac{y_i}{y_o} = \frac{-S_i}{S_o} = \frac{-f_o}{x_o} = \frac{-x_i}{f_i}$$

- **Longitudinal Magnification:**

$$M_L = \frac{\Delta x_i}{\Delta x_o}$$



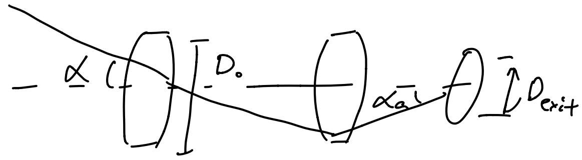
for small object length:

$$\boxed{M_L = \frac{-f^2}{x_o^2} = -M_T^2}$$

- **System of Lens:**

  - Image of first is used as object of 2nd
  - $M_T = M_1 \times M_2 \times \ldots \times M_n.$

Astronomical Telescope



- $M_\alpha = \dfrac{-f_{objective}}{f_{eyepiece}} = \dfrac{\alpha_a}{\alpha}$

- Tube length $(d) = f_o + f_e$

- $x_i = \dfrac{f^2}{x_o} = \dfrac{-f_e^{\,2}}{f_o} = \dfrac{f_e}{M_\alpha}$

- image is <u>inverted</u>

- $\dfrac{D_{exit}}{D_{entrance}} = \dfrac{D_o}{D_e} = \dfrac{-f_e}{x_o} = \dfrac{-f_e}{f_o} = \dfrac{1}{M_\alpha}$

Galilean Telescope

- Same formulas, but image is upright, and 2^{nd} lens is <u>diverging</u>

Telescope Resolution

$$\theta_{res} = \frac{1.22\lambda}{d}$$ d = entrance pupil diametre.

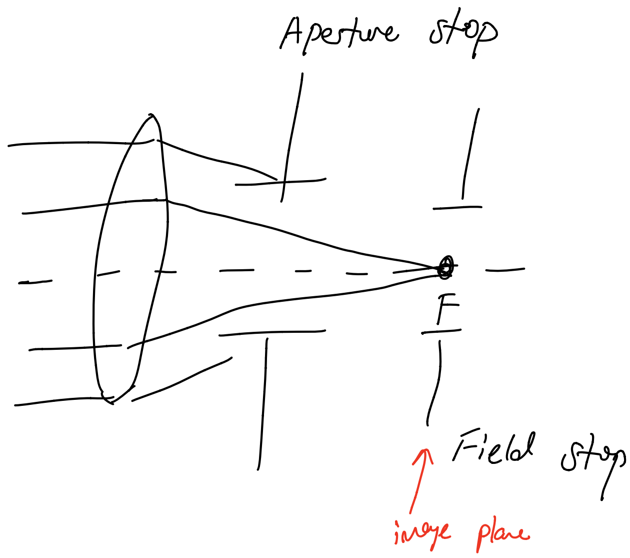
$$\tan\theta_{res} = \frac{\text{feature seperation}}{\text{Object distance}}$$

Apertures or Stops

- A stop limits the light rays entering the system

- Aperture stop: limits on-axis rays (variable aperture: iris)
  Field stop: limits off-axis rays

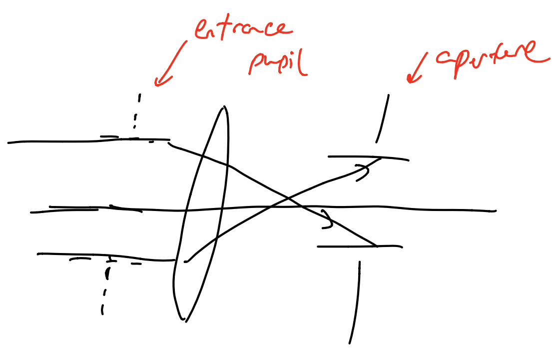
- Apperture stop size ↓:
  - reduce aberration
  - increase blur due to diffraction
  - increase depth of focus / field
  - decrease illumination



- field stop size ↓:
  - decrease illumination
  - decrease field of view

- Both stops can be a actual stop, or one of the lenses or mirrors of the system

Entrance Pupil



- image of the aperture stop

- Size of the entrance pupil determines the amount of light reaching the image plane.

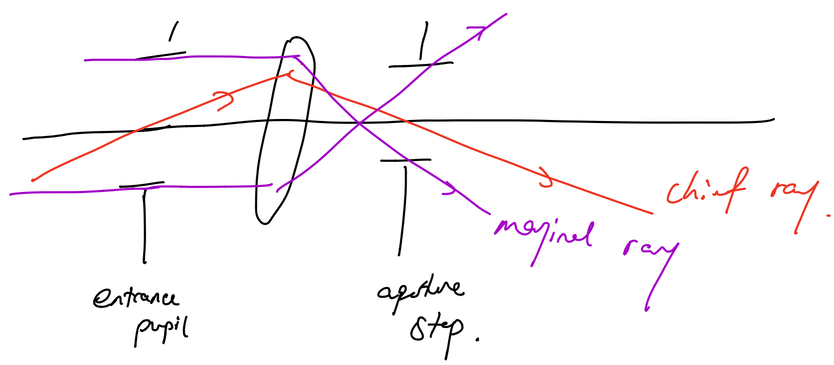
$$\left(\frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{f} \right)$$

Chief Ray

- ray passing through the centre of aperture stop
- For axial object, chief ray defines the centre of a defocused or the aberrated (blur) image. ⇒ extend of image

Marginal Rays

- go towards the edge of the entrance pupil, edge of aperture stop, appear to come from edge of exit pupil
- Define blur



f-Number

- $f_{\#} = \dfrac{f_0}{D}$ ← diameter of entrance pupil

$I \propto \dfrac{1}{f_{\#}^{2}}$ for same exposure time

$I \propto D^2, \quad I \propto f_0^{-2}$

$I \propto (\text{image area})^{-2}$

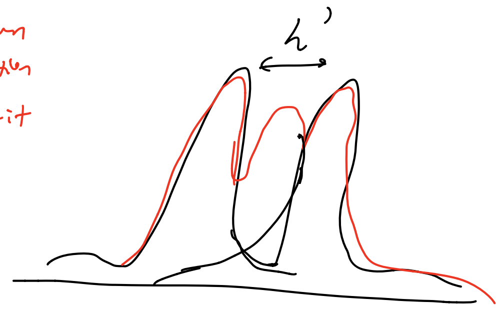
- $f/_{8}$ means $f_{\#} = 8$

- For constant brightness, exposure time must be adjusted upwards as $f_\#^2$ increase $\Rightarrow t \propto f_\#^2$ (increase $f_\#$ by 2, increase t by 2^2)

- Smaller $f_\#$ = brighter image & small diffraction blur

- Diffraction $\propto \frac{1}{D_{entrance}}$ $\Rightarrow$ for small apertures, diffraction dominates

- $\theta_{res} = \frac{1}{2}\theta_{blur} = 1.22\frac{\lambda}{D}$ minimum resolution limit



- $h' = 1.22\,\lambda\frac{f_o}{D}$

  ↳ Linear extent of detail resolved in images.

- Size of diffraction image = $2.44\frac{\lambda f_o}{D}$

Optical lens system

- Thin Lens Equation:

$$\frac{n_m}{S_o} + \frac{n_m}{S_i} = (n_L - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{n_m}{f}$$

$$f_o = f_i = f$$

$$\frac{n_m}{S_i} + \frac{n_m}{S_o} = \frac{n_m}{f}, \quad x_i = \frac{f^2}{x_o}$$

$$M_T = \frac{y_i}{y_o} = \frac{-S_i}{S_o} = \frac{-f}{x_o} = \frac{-x_i}{f}$$

$$x_o x_i = f^2$$

- For 2 thin lenses / one thick lens:

$$S_{o_2} = d - S_{i_1}$$

$$M_T = M_1 \times M_2$$

$$\frac{1}{f_{sys}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2 - \frac{d}{n_{air}} \mathcal{D}_1 \mathcal{D}_2$$

2 thin lenses seperated by air

$$bfl = f_i \left[1 - \frac{d}{f_{i_1}}\right] = f_i + h_2$$ ⇒ back focal length, V_2 to F_i

$$ffl = f_o \left[1 - \frac{d}{f_{o_2}}\right] = f_o - h_1$$ ⇒ front focal length, V_1 to F_o

$$\frac{n_m}{S_i} + \frac{n_m}{S_o} = \frac{n_m}{f} = \frac{n_L - n_m}{R_1} - \frac{n_L - n_m}{R_2} + \frac{(n_L - n_m)^2 d}{n_L R_1 R_2} = \mathcal{D}$$

$$h_2 = \frac{-f(n_L - 1)d}{R_1 n_L} = \frac{-fd}{f_{i_1}}$$ = 2nd / back principle plane

$$h_1 = \frac{-f(n_L - 1)d}{R_2 n_L} = \frac{fd}{f_{o_2}}$$ = 1st / front priciple plane

$$\frac{y_i}{y_o} = \frac{-S_i}{S_o} = \frac{-x_i}{f} = \frac{-f}{x_o} \; ; \; \frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{f_o} \, , \; x_i x_o = f_o f_i = f^2$$

• Interference

$$I_T = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos\delta$$

- $\cos\delta < 0$ for destructive interference $\Rightarrow \delta = \frac{\Delta}{\lambda}2\pi,\ \Delta = 0, \lambda, 2\pi \dots$

  ↳ min at $\delta = \pm\pi, \pm 3\pi \dots$

- $\cos\delta < 0$ for constructive interference $\Rightarrow \delta = \frac{\Delta}{\lambda}2\pi,\ \Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \dots$

  ↳ max at $\delta = 0, \pm 2\pi, \pm 4\pi \dots$

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\delta),\quad \delta = \alpha_1 - \alpha_2$$

• phase Difference

lag distance

$$\delta = \frac{\Delta x}{\lambda}2\pi = \frac{2\pi}{\lambda_0}n\Delta x = k_0 n\Delta x \text{ rad.}$$

Optical path difference (OPD) $= \Delta \sum_i n_i x$

• Adding Coherent Waves $\Rightarrow \delta = 0$

Irradiance $\propto$ (Amplitude)^2

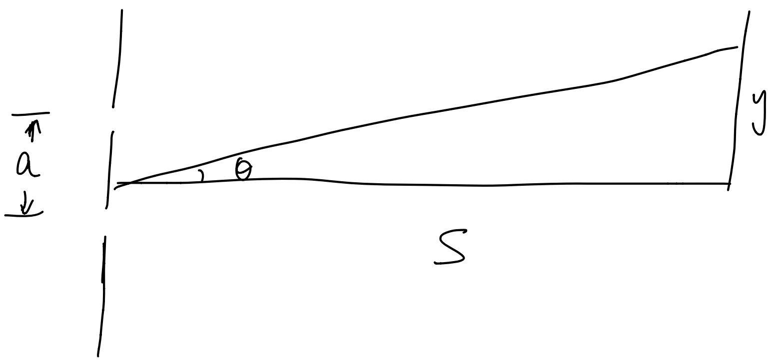
$$\Rightarrow I = N^2 I_0$$

- ## 2 Slits Interference

For equal, coherent input: $I_{1+2} = 4I_0 \cos^2\frac{\delta}{2}$

$I_{max} = 4I_0$ for $\delta = 2m\pi$

$I_{min} = 0$ for $\delta = (2m+1)\pi$.



$\delta = k\Delta x = \frac{2\pi\Delta x}{\lambda} = \frac{ay\,2\pi}{s\lambda}$

for small θ

$\Rightarrow$ $\downarrow$ seperation, fringe width $\uparrow$

$\uparrow$ distance, fringe width $\uparrow$

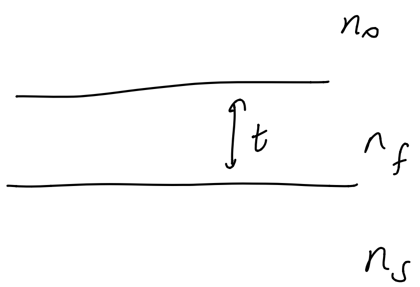
$\uparrow$ seperation, decrease δ

$y_{max} = \frac{m\lambda s}{a}$, $m = 0, \pm 1, \pm 2 \ldots$ $\Rightarrow$ $a\sin\theta = m\lambda$

Seperation of max: $\Delta y_{max} = \frac{\lambda s}{a}$

- ## Thin Film

fast $\rightarrow$ slow $\rightarrow$ phase change $= \pi$ $\Rightarrow \Delta r_x = \frac{\lambda}{2}$



for $n_o < n_f < n_s$ $\Rightarrow$ no phase difference

$\hookrightarrow \Delta r_x = 0$

for $n_o < n_f$, $n_f > n_s$ $\Rightarrow \Delta r_x = \frac{\lambda}{2}$, $\delta = \pi$.

$\Rightarrow$ For constructive: $OPD + \Delta r_\lambda = m\lambda$; $OPD = 2n_f t$

For destructive: $OPD + \Delta r_\lambda = (m+\frac{1}{2})\lambda$

- **Anti-Reflection Coating**

  - Amplitude Condition:

$$E_{0_1} = E_{0_i}\left[\frac{n_f - 1}{n_f + 1}\right] = E_{0_2} \approx E_{0_i}\left[\frac{n_s - n_f}{n_s + n_f}\right]$$

in air: $\boxed{n_f = \sqrt{n_s}}$; in other medium: $n_f = \sqrt{n_0 n_s}$

  - Path Condition:

$$n_f t = \frac{\lambda_0}{4} \quad (\text{destructive for } m = 1)$$

- **Michelson Interferometer**

max: $m\lambda = 2d\cos\theta$ min: $2d\cos\theta = (m+\frac{1}{2})\lambda$

$\Rightarrow$ for a constant viewing angle:

$$2\Delta d = \Delta n \lambda$$

- **Multiple Slits**

Fraunhofer Condition: $R^2 > a^2/\lambda$

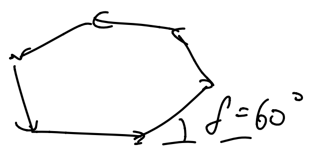
For N Slits: $\boxed{\delta = \frac{d\sin\theta}{\lambda} 2\pi} = 2m\pi \Rightarrow OPD = d\sin\theta = m\lambda$.

for constructive

destructive: $OPD = d\sin\theta = (m+\frac{1}{2})\lambda$

ex: For 6 constructive:

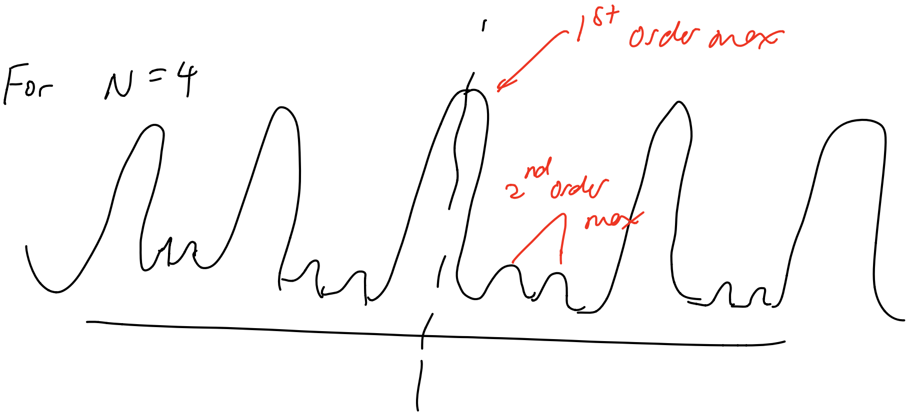
For 6 destructive:



$$\delta = \frac{2m\pi}{N} \quad \text{for min.}$$

$$I = I_0\left(\frac{\sin(N\alpha)}{\sin\alpha}\right)^2, \quad \alpha = \frac{\delta}{2}$$

For $N=4$



- # of 2nd order max = $N-2$
- # of min = $N-1$
- As $N \uparrow$, 1st order max get narrower
- As $N \uparrow$, height of 2nd order max $\downarrow$.

- Diffraction

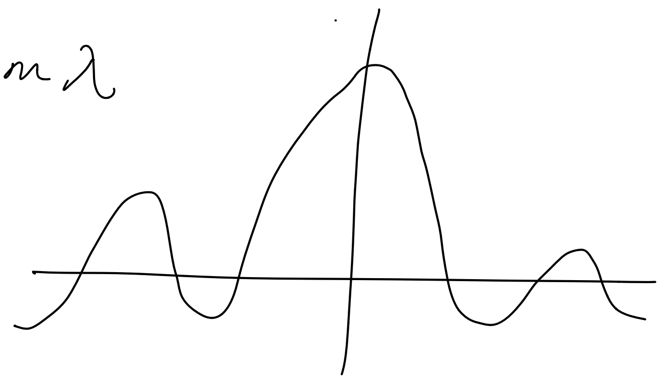
- Fraunhofer diffraction condition: $R^2 > b^2/\lambda$, b = Slit width

- Single slit:

$$\delta = \frac{2\pi}{\lambda} b \sin\theta$$

For minima: $\delta = 2m\pi$, $m \neq 0$

$$b \sin\theta = m\lambda$$



$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

$$I(\theta) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2$$

- Multiple Slit Diffraction

$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2, \quad \alpha = \frac{\pi a \sin\theta}{\lambda}.$$

# of friges in central max:

↳ Find edge of central diffraction $\boxed{\text{max.}}$ $(m=1)$

$$\sin\theta = \frac{\lambda}{b}$$

↳ Then, find interference min:

$$\sin\theta = (m+\tfrac{1}{2})\frac{\lambda}{a}$$

↳ Solve for m

↳ # of fringes $= 2m+1$

- Missing max fringe when θ_m in N slits $= \theta_n$ in single slit.